Module 3

DC to DC Converters

Lesson
25
Design of Transformer for Switched Mode Power Supply (SMPS) Circuits

After completion of this lesson the reader will be able to:

(i) Explain the underlying principles behind the design of a high frequency transformer and inductor.
(ii) Do a preliminary design of a high frequency transformer for some popular configurations of SMPS circuits.
(iii) Do a preliminary design of a high frequency inductor.
(iv) Estimate the size of an SMPS transformer of some given VA rating.

Transformers are required for galvanic isolation between input and output voltages and for voltage and current scaling. It also helps in optimizing the device voltage and current ratings. The switches, diodes and other circuit elements on the high voltage side of the transformer are subjected to higher voltages but only lower currents. Similarly the devices put on the low voltage side are subjected to less voltage stress but higher current stress. The dc-to-dc buck converter shown in Fig. 25.1, which is used to get a low voltage output from a high input dc voltage illustrates this point clearly. The circuit in Fig. 25.1(a) uses a step down transformer with proper turns ratio and has the advantages discussed above. On the other hand the switch and diode and the filter inductor in Fig. 25.1(b) need to withstand both input side voltage and output side current. Also, the switch in case (b) will be constrained to operate in a narrow range, which may cause lesser accuracy in output voltage control.

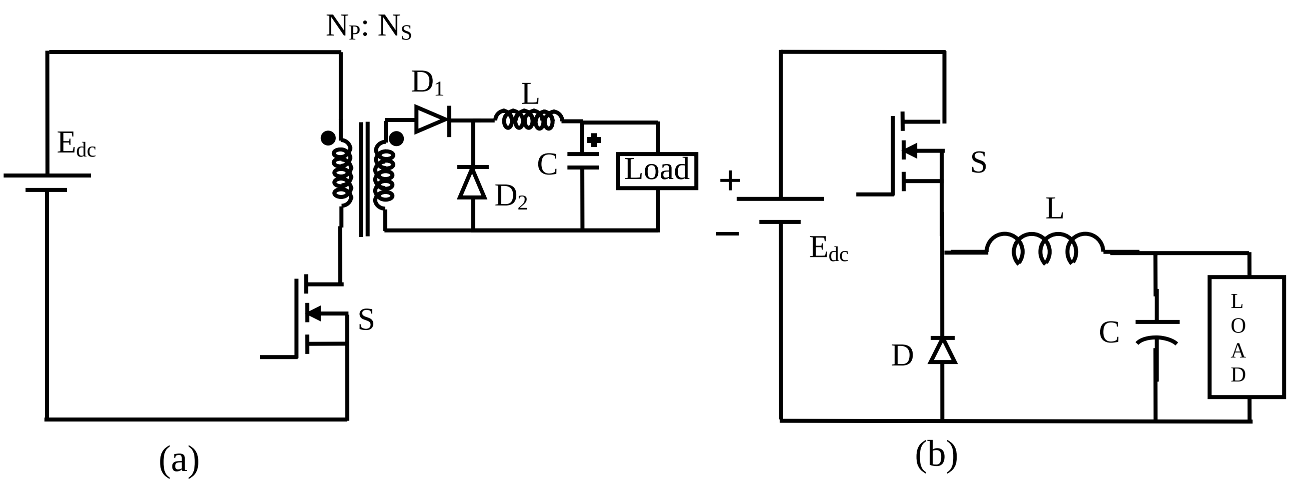

Fig. 25.1: DC to DC buck converters: (a) Isolated type (b) Non-isolated type

Transformers used in switched mode power supply circuits are significantly different from the power transformers that are used in utility ac supply system. Following are the important differences:

(i) The input and output voltages and currents of a SMPS transformer are mostly non-sinusoidal, whereas the transformers connected to utility ac supply are almost always subjected to sinusoidal voltages and currents.
(ii) The currents and voltages of SMPS transformer are of very high frequency where as utility type transformers are subjected to low frequency supply voltages.
(iii) SMPS transformers generally handle much smaller power than the utility transformer.

SMPS transformer-core, because of high frequency operation, is generally made of hard magnetic material like ferrites whereas the low frequency power transformers mostly use soft magnetic material like silicon steel. Ferrites have very high ohmic resistance and the area enclosed under the hysteresis loop of their B-H magnetization curve is significantly lower than that of silicon steel. As a result, even at very high frequency operation, the hysteresis and eddy current losses are low. [Low hysteresis loss is due to less B-H loop area and low eddy current loss is due to very high resistivity of the core material.] The ferrites have low magnetic permeability (typical value of relative permeability is around 100) and low saturating value of flux density (typical value is around 0.4 Tesla) that are considerably less than that of silicon steel. Ferrites are also brittle and fragile. The efforts are on to search for alternatives to ferrites that may have higher permeability, may handle higher flux density and may be more rugged.

The fundamental principles concerning emf generation etc. in SMPS-transformers and power transformers are identical and hence, in this lesson, many concepts of conventional transformer design have been borrowed.

25.1 Recapitulation of Governing Equations for Utility Transformer

In case of sinusoidal flux of peak magnitude 'ϕ_m' and frequency 'f' linking the transformer windings, the emf generated per turn of the winding will have a rms magnitude 'E_t' given by:

$$E_t = 4.44\, f\, \phi_m \qquad \text{---}(25.1)$$

The peak flux through the core is the product of peak flux density (B_m) and the core area (A_c), i.e.,

$$\phi_m = B_m A_c \qquad \text{--- }(25.2)$$

The windings are placed around the core and are accommodated in the window of the transformer. The transformer window area (A_w) is related with the winding's current rating and the number of turns. For a single-phase transformer the relation between them is given by:

$$A_w\, k_w\, \delta = 2\, N\, I \qquad \text{--- }(25.3)$$

,where k_w is the window utilization factor and δ is the current density through the cross-sectional area of the transformer windings. Window utilization factor, roughly varies between 0.35 to 0.6 and is dependent on the insulation requirements of the windings. A typical figure for the current density through copper conductors of naturally cooled transformers is $3\text{X}10^6$ amps per square meter. If the current density through primary and secondary windings is taken identical, they occupy equal window-space of the transformer. Some times the current densities through the two windings may differ depending on their physical ability to dissipate heat. The VA rating of a single phase transformer (= N E_t I) can now be found from the above equations as:

$$\text{VA rating} = 2.22\, f\, B_m\, \delta\, k_w\, A_c\, A_w \qquad \text{----------------------------}(25.4)$$

For the given operating frequency (f) the product '$A_c\ A_w$', known as area product is roughly proportional to the VA rating of the transformer as other parameters have nearly fixed magnitudes.

25.2 Derivation of Design Equations for SMPS Transformer

The nature of voltage and flux waveforms in SMPS transformers is different from that of utility transformer. Moreover SMPS circuits of different topologies generate different kinds of winding voltages (and hence the flux-linked waveforms) and need to be considered separately.

In this section some representative voltage and flux waveforms have been taken up and through them the transformer design procedure has been illustrated.

25.2.1 Transformer with Square-Wave Voltage and Bipolar Flux

Fig. 25.2 shows the typical winding voltage and core-flux waveform produced by one of the popular SMPS topologies that utilizes a H-bridge converter to get high frequency ac voltage from the dc input. The primary side of the SMPS transformer is connected to the H-bridge output and the secondary side voltage is rectified and filtered to get regulated dc output voltage of desired magnitude. The transformer windings carry bi-direction current and the flux linking the windings is also bipolar. The input dc bus voltage is unregulated and often varies over a large range. The duty ratio 'D' of the switches is controlled within $0<D<0.5$ to regulate the output voltage. The mean of the rectified secondary side voltage, under steady state and after accounting for voltage drops in the rectifier diode and filter inductor, equals the desired load voltage and can be assumed fixed to the output voltage 'V_o'. However under dynamic condition, which may arise due to sudden change in load or supply voltage, the mean (dc) output voltage on the secondary side may be significantly higher than its steady state magnitude. For calculation of peak flux in the core, the worst-case condition will correspond to maximum duty ratio (D=0.5) and maximum magnitude of input voltage. The worst-case current through the windings will correspond to maximum duty ratio (D=0.5) and peak magnitude of output (load) current. Now the transformer may be designed as per the design steps given below:

(i) <u>Determination of primary to secondary turns ratio (N_P/N_S):-</u> This can be found from the knowledge of operating range over which the input dc voltage may vary. Let the input voltage vary from V_{min} to V_{max}. With minimum input voltage 'V_{min}' and duty ratio 'D' = 0.5, the magnitude of square-shaped secondary side voltage should equal $(V_o + V_R)$, where V_R is the estimated voltage drop in the transformer winding, output rectifier and filter circuit under maximum load condition. The transformer turns ratio can thus be estimated to $N_P/N_S = V_{min}/(V_o + V_R)$. The actual number of turns in the windings will be found as shown below in step (v).

(ii) <u>Determination of peak magnitude of flux in the transformer core:</u> As per above discussion, the maximum flux in the core will correspond to a square wave voltage of magnitude V_{max} across the primary winding (refer to Fig. 25.2 with D=0.5). The frequency of voltage waveform 'f'(=1/T) is same as the frequency at which the converter switches are turned on and is fixed beforehand. Now by simple integration of the square wave voltage waveform, the peak flux 'ϕ_m' is related to the input voltage as, $V_{max} = 4.0\ f\ \phi_m\ N_P = 4.0\ f\ B_m\ A_c\ N_P$ --------------------------(25.5)

(iii) Determination of winding current rating and requirement of window area: Let 'I_{om}' be the peak expected load current. The secondary winding of the transformer should be rated to supply this current. Most SMPS circuits, with low magnitude of output voltage, have a center-tapped secondary winding followed by a mid-point rectifier circuit realized using two diodes (instead of bridge rectifier having four diodes). This results in only one diode voltage drop during rectification, unlike two diode drops for the bridge rectifier circuit. For SMPS with low output voltage, saving one diode drop can result in significant increase in the efficiency. For this same reason, the diodes used on the secondary side are Schottky diodes having low on-state voltage drop.

Each half of the center-tapped secondary winding requires N_S turns as determined in (i) above and they carry the load (dc) current only in alternate half cycles. Thus the rms current rating of each half equals $\frac{I_{om}}{\sqrt{2}}$ and the net copper cross-sectional area required for the secondary winding is $\frac{\sqrt{2}N_S I_{om}}{\delta}$, where δ is the current density (as described in relation to Eqn.25.3). If the secondary was not center-tapped, the rectifier used would be bridge type and the copper area for the secondary would have been just $\frac{N_S I_{om}}{\delta}$. The primary side carries the reflected secondary current and the required copper area for primary would equal $\frac{N_S I_{om}}{\delta}$. The total window area requirement for the transformer can now be given as:

$$A_w k_w = \frac{N_S I_{om}}{\delta}\left(1+\sqrt{2}\right) \text{ ------------ } (25.6),$$

where A_w is the window area and k_w is the window utilization factor (as discussed in Sec.25.1).

(iv) Expression for VA rating of the transformer: Combining Eqns. (25.5) and (25.6) one gets,

$$V_{max}\frac{N_S}{N_P}I_{om}\left(1+\sqrt{2}\right) = 4fB_m\delta k_w A_c A_w \text{ --------------------------} (25.7)$$

Using relations derived in (i) above, Eqn.25.7 may be rewritten as:

$$V_o I_{om} K_1 K_2\left(1+\sqrt{2}\right) = 4fB_m\delta k_w A_c A_w \text{ --------------------------} (25.8)$$

where $K_1 = \frac{V_{max}}{V_{min}}$, a factor allowing for input voltage variation and $K_2 = \frac{V_0 + V_R}{V_o}$, a factor coming due to voltage drop in rectifier diode, filter inductor etc. V_o I_{om} is the peak output power from the SMPS. The factor $\left(1+\sqrt{2}\right)$ on the L.H.S. of Eqn.27.8 will become 2.0 if the secondary winding is not center-tapped.

(v) Selection of transformer core and determination of number of turns in the windings:

Knowing the area product 'A_c A_w', as given by Eqn.25.8, the appropriate transformer core is to be selected from the core-manufacturer's catalog. Once the area product matches, the details of other dimensions of the transformer core are found from the catalog. Knowing window area (A_w) and core area (A_c), the number of turns in the windings can be decided using Eqns. Like (25.5) or (25.6).

25.2.2 Transformer with Unipolar Flux

Many switched mode power supply circuits use only one controlled switch (like the forward converter discussed in Lesson-23). The winding current and core-flux for most of these transformers are unidirectional. Fig. 25.3 shows the typical winding voltage along with the corresponding core-flux waveform for a forward converter. As shown in Lesson-23, when the forward converter switch is turned on the primary winding is subjected to input dc voltage. As soon as the primary winding is turned-off, the tertiary winding starts conducting and the voltage across primary goes negative with a magnitude that equals the product of input voltage and the turns ratio between the primary and tertiary windings. The maximum duty ratio (D_{max}) of the switch is also limited by the turns ratio between the primary and tertiary winding to allow resetting of the transformer flux (as given in Sec.23.4 of Lesson-23). The maximum input voltage (V_{max}), switching frequency 'f'(=1/T) and the maximum duty ratio (D_{max}) are related with the peak magnitude of core-flux is calculated as

$$V_{max}\, D_{max} = f\, \phi_m\, N_P = f\, B_m\, A_c\, N_P \qquad \text{--(25.9)}$$

Eqn.25.9 may be compared with Eqn.25.5 for a typical value of D_{max} = 0.5 (which corresponds to the case when primary and tertiary windings have identical number of turns). Because of unipolar nature of flux the utilization of core (in terms of emf generation) is poorer here.

The primary to secondary turns ratio (N_P/ N_S) for the forward converter can be estimated as done previously for the H-bridge converter. Accordingly, $N_P/N_S = V_{min}\, D_{max} /(V_o + V_R)$, where V_o is the required output voltage and V_R denotes the voltage drop in output rectifier and filter circuit.

The maximum rms current through the secondary winding can be equated to $I_{om}\sqrt{D_{max}}$ and the window area (A_w) requirement is given by $A_w k_w = \dfrac{2N_S I_{om}\sqrt{D_{max}}}{\delta}$ -----------------(25.10)

From Eqn.25.9 and 25.10, the VA rating of the transformer is given as: $V_{max}\dfrac{N_S}{N_P} I_{om}\left(D_{max}\right)^{1.5} = 0.5 f B_m \delta k_w A_c A_w$, which may be rewritten as

$$V_o I_{om} K_1 K_2 \sqrt{D_{max}} = 0.5 f B_m \delta k_w A_c A_w \qquad \text{---(25.11)}$$

Eqn.25.11 is similar to Eqn.25.8 above. The symbols used also denote the same. Knowing the window area, the transformer core selection and other designs are done as described above in connection with the H-bridge topology. The extra tertiary winding of a forward converter transformer carries only magnetization current, which is a quite small and even a thin gauge wire will serve the purpose. However, with the addition of tertiary winding the insulation requirement of the transformer increases significantly and hence the window utilization factor (k_w) becomes low.

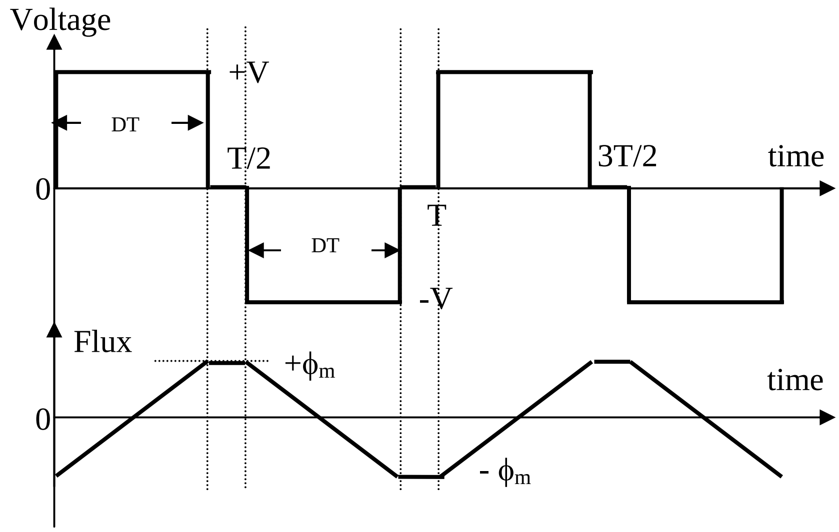

Fig. 25.2: Winding voltage and core-flux waveforms for a H-bridge type SMPS supply

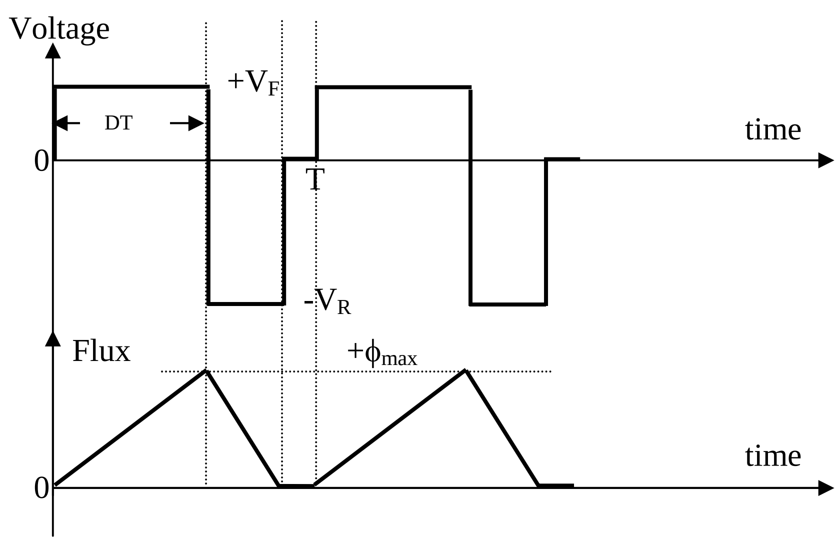

Fig. 25.3: Winding voltage and core-flux waveforms for a forward type SMPS supply

25.2.3 Design of Inductor-Transformer

The fly-back type SMPS circuits use a different kind of transformer, which as indicated in Lesson-22, may be more appropriately called as inductor-transformer. Such a transformer is more like two coupled inductors. These two coupled-inductors don't conduct simultaneously, unlike the two coupled-windings of a normal transformer. Also, the inductance needs to have a finite magnitude so that current can build through it during each high frequency cycle and the inductor may store the desired magnitude of energy. The windings of an inductor-transformer facilitate energy storage in the magnetic field whereas the windings of an ideal transformer (having infinitely large permeability 'μ' of the core) cannot be used for storing energy as energy density equals $0.5\frac{B^2}{\mu}$. For finite magnitude of flux density 'B', the magnitude of 'μ' should be small to have higher energy per unit volume. 'μ' and magnetic reluctance have inverse relation, as 'μ' decreases the reluctance increases. For a practical inductor the reluctance of its flux-path should not be zero.

For an inductor, working in the linear region of the core's magnetization, the following relation holds good between inductance (L), reluctance (R) and the number of turns (N) of the inductor: $L = \frac{N^2}{R}$. However a practical inductor still requires a good core with high permeability to increase (i) coupling between the windings, (ii) to guide the flux path and hence decrease the stray magnetic field lines and (iii) to keep the inductor size small. However to keep the reluctance of the flux-path at the desired value, an appropriate length of air-gap is introduced in the flux path. Fig.25.4 shows a double 'E' core with windings put around the central limb. After the windings are placed in position, a non-magnetic material (like, paper) is inserted between the faces of the core and the two 'E's of the core are clamped together. The non-magnetic material acts like air-gap in the core. A preferred way of creating air-gap may be to grind some length from only the central limb of the core. If 'l_g' is the length of air-gap in the core, the inductance (L) can be expressed as:

$$L = \frac{N^2 A_c \mu_0}{l_g} \quad \text{--} \ (25.12)$$

where A_c is the area of the core's limb on which the windings have been placed and μ_0 is the permeability of air-gap. In the above expression for inductance, the fringing effect of the flux and the reluctance of the flux path through magnetic core have been neglected.

The core material should not saturate with the peak expected current (I_p) in the inductor. The peak flux density in the core (B_m) can be related with the peak magnitude of current as

$$LI_p = NA_cB_m \quad \text{--} \ (25.13)$$

Knowing the current shape through the inductor, one calculates its rms magnitude ($I_{p,rms}$) and determines the window area required as $A_w k_w = \frac{NI_{p,rms}}{\delta}$ ------------------------------(25.14)

Combining Eqns.25.13 and 25.14, one gets

$$LI_p I_{p,rms} = B_m \delta k_w A_c A_w \text{--} \ (25.15)$$

Eqn.25.15, gives the area product from which rest of the design can be proceeded as in the case of transformer design shown above. LHS of Eqn.25.15 is indicative of the energy holding capacity of the inductor (some what like VA rating of the transformer discussed above). Should there be a couple winding (an inductor-transformer) the area product expression needs to be modified to include the window space requirement of the secondary winding as well.

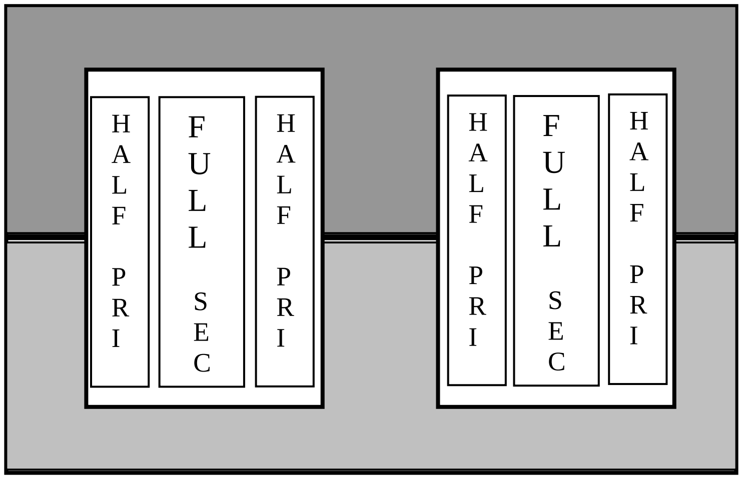

Fig. 25.4: A typical SMPS transformer with a double 'E' type ferrite core and interleaved primary and secondary winding

25.3 Transformer Winding

Often sandwiched type windings (as shown in Fig.25.4, where the secondary winding is sandwiched between two halves of the primary) are used to reduce leakage inductance of the windings. Sandwiching increases the insulation requirement between the windings.

For very high frequency applications, it may be preferred to use ribbon-conductor or copper foil in place of solid circular conductors. This helps in better utilization of winding's copper as high frequency current is effectively limited to the surface of the conductor.

Many applications require grounded shields around the windings to reduce electro-magnetic interference (EMI) caused by the SMPS transformers. As discussed in this lesson the SMPS transformers often carry very high frequency ripples. These shields are essentially 3/4th turn of a metallic foil put around the windings. There should be proper insulation between the shield and

Quiz Problems

(1) For a high frequency transformer the relation between the transformer size and frequency of voltage waveform can be given as:

(a) Size increases with frequency
(b) Size decreases with frequency
(c) Core size reduces but copper weight increases with increase in frequency
(d) Size is independent of frequency

(2) The assembly of fly-back and forward type transformer cores may differ in the following sense:

(a) Air-gap is inserted in fly-back type but it is undesirable for forward type.
(b) Air-gap in the flux path is undesirable for both types
(c) Only forward type must have a suitably length of air-gap
(d) Little air-gap is deliberately put for both transformers

(3) Transformers of forward type and H-bridge type SMPS circuits of identical VA rating and frequency differ in the following sense:

(a) The forward type transformer will be bigger
(b) The H-bridge circuit will require bigger transformer
(c) They will be of identical size
(d) Only the window area of H-bridge transformer will be bigger

(4) The size of SMPS transformers operating over large input voltage range will compare with similar rated transformer operating over a narrower input voltage range in the following manner:

(a) Larger input voltage range will require larger transformer
(b) Larger voltage range requires smaller transformer
(c) Size remains independent of voltage range

(Answers: 1-b, 2-a, 3-a, 4-a)